

Quantitative Literacy: Thinking Between the Lines

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Chapter 4: Personal Finance

Chapter 4: Personal Finance

Lesson Plan

- ▶ Saving money: The power of compounding
- ▶ Borrowing: How much car can you afford?
- ▶ Savings for the long term: Build that nest egg
- ▶ Credit cards: Paying off consumer debt
- ▶ Inflation, taxes, and stocks: Managing your money

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4.2 Borrowing: How much car can you afford?

Learning Objectives:

- ▶ Understand installment loans
- ▶ Calculate Monthly payment of a fixed loan
- ▶ Calculate amount borrowed
- ▶ Understand Amortization table and equity
- ▶ Understand mortgage options: fixed-rate mortgage vs. Adjustable-rate mortgage
- ▶ Compute the monthly payment for a mortgage

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4.2 Borrowing: How much car can you afford?

- ▶ With an **installment loan** you borrow money for a fixed period of time, called the term of the loan, and you make regular payments (usually monthly) to pay off the loan plus interest accumulated during that time.
- ▶ The amount of payment depends on three things:
 1. the amount of money we borrow (the principal)
 2. the interest rate (or APR)
 3. the term of the loan

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4.2 Borrowing: How much car can you afford?

- ▶ Monthly payment formula

$$\begin{aligned} & \text{Monthly payment} \\ &= \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)} \end{aligned}$$

where t is the term in months and $r = APR/12$ is the monthly rate as a decimal.

- ▶ **Example (College Loan):** You need to borrow \$5,000 so you can attend college next fall. You get the loan at an APR of 6% to be paid off in monthly installments over three years. Calculate monthly payment.

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4.2 Borrowing: How much car can you afford?

- ▶ **Solution:** The interest rate (or APR): APR of 6%

The monthly rate as a decimal is:

$$r = \text{Monthly rate} = \frac{APR}{12} = \frac{0.06}{12} = 0.005$$

We want to pay off the loan in three years, so we use a term of $t = 3 \times 12 = 36$ in the monthly payment formula:

$$\begin{aligned} \text{Monthly payment} &= \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)} \\ &= \frac{\$5000 \times 0.005 \times 1.005^{36}}{(1.005^{36} - 1)} = \$152.11 \end{aligned}$$

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4.2 Borrowing: How much car can you afford?

- ▶ Suppose you can afford a certain monthly payment, how much you can borrow to stay that budget.

Companion monthly payment formula

Amount borrowed

$$= \frac{\text{Monthly payment} \times ((1 + r)^t - 1)}{(r \times (1 + r)^t)}$$

- ▶ **Example (Buying a car):** We can afford to make payments of \$250 per month for three years. Our car dealer is offering us a loan at an APR 5%. For what price automobile should we be shopping?

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4.2 Borrowing: How much car can you afford?

Solution: The monthly rate as a decimal is:

$$r = \text{Monthly rate} = \frac{0.05}{12} = 0.0042$$

Three years is 36 months, so we use a term of $t = 36$ in the companion monthly payment formula:

$$\begin{aligned} & \text{Amount borrowed} \\ &= \frac{\text{Monthly payment} \times ((1 + r)^t - 1)}{(r \times (1 + r)^t)} \\ &= \frac{\$250 \times ((1 + 0.0042)^{36} - 1)}{(0.0042 \times (1 + 0.0042)^{36})} = \$8,341.43 \end{aligned}$$

We should shop for cars that cost \$8,341.43 or less.

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4.2 Borrowing: How much car can you afford?

- ▶ **Amortization table (schedule):** shows for each payment made the amount applied to interest, the amount applied to the balance owed, and the outstanding balance.
- ▶ If you borrow money to pay for an item, your **equity** in that item at a given time is the part of the principal you have paid.
- ▶ **Example:** Suppose you borrow \$1,000 at 12% APR to buy a computer. We pay off the loan in 12 monthly payments. Make an amortization table showing payments over the first five months. What is your equity in the computer after five payments?

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4.2 Borrowing: How much car can you afford?

► **Solution:**

The monthly rate $r = \frac{12\%}{12} = 1\% = 0.01$ as a decimal.

The monthly payment formula with $t = 12$:

$$\begin{aligned} & \text{Amount borrowed} \\ &= \frac{\text{Monthly payment} \times ((1 + r)^t - 1)}{(r \times (1 + r)^t)} \\ &= \frac{\$1000 \times ((1 + 0.01)^{12} - 1)}{(1.01^{12} - 1)} \\ &= \$88.85 \end{aligned}$$

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4.2 Borrowing: How much car can you afford?

- Make our **1st payment**: the outstanding balance is \$1000

Interest = 1% of \$1000 = \$10

\$88.85 – \$10 = \$78.85 to interest, and the outstanding balance.

Balance owed after 1 payment

$$= \$1000 - \$78.85 = \$921.15$$

- Make our **2nd payment**: the outstanding balance is \$921.15

Interest = 1% of \$921.15 = \$9.21

\$88.85 – \$9.21 = \$79.64 to interest, and the outstanding balance.

Balance owed after 2 payments

$$= \$921.15 - \$79.64 = \$841.51$$

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❑ If we continue in this way, we get the following table:

Payment number	Payment	Applied to interest	Applied to balance owed	Outstanding balance					
				\$1,000.00					
1	\$88.85	1% of \$1000.00 = \$10.00	\$78.85	\$921.15					
2	\$88.85	1% of \$921.15 = \$9.21	\$79.64	\$841.51					
3	\$88.85	1% of \$841.51 = \$8.42	\$80.43	\$761.08					
4	\$88.85	1% of \$761.08 = \$7.61	\$81.24	\$679.84					
5	\$88.85	1% of \$679.84 = \$6.80	\$82.05	\$597.79					
6	\$88.85	1% of \$597.79 = \$5.98	\$82.87	\$514.92					

❑ Equity after five payments = \$1000 – \$514.92 = \$485.08.



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4.2 Borrowing: How much car can you afford?

- ▶ **Home Mortgage:** a loan for the purchase of a home.
- ▶ **Example (30-year mortgage):** You decide to take a 30-year mortgage for \$300,000 at an APR of 9%. Find the total interest paid.
- ▶ **Solution:** The monthly payment rate: $r = \frac{APR}{12} = \frac{0.09}{12} = 0.0075$.
 - ▶ The loan is for 30 years: $t = 30 \times 12 = 360$ months in the monthly payment formula:

$$\begin{aligned} \text{Monthly payment} &= \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)} \\ &= \frac{\$300,000 \times 0.0075 \times 1.0075^{360}}{(1.0075^{360} - 1)} = \$2413.87 \end{aligned}$$

- ▶ Total amount paid = $360 \times \$2413.87 = \$868,993.20$
- ▶ Total interest paid = $\$868,993.20 - \$300,000 = \$568,993.20$

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4.2 Borrowing: How much car can you afford?

- ▶ **Fixed-rate mortgage:** keeps the same interest rate over the life of the loan.
- ▶ **Adjustable-rate mortgage (ARM):** the interest may vary over the life of the loan.

- ▶ **Example (comparing monthly payment):** Fixed-rate mortgage and ARM.

We want to borrow \$200,000 for a 30-year home mortgage. We have found an APR of 6.6% for a fixed-rate mortgage and an APR of 6% for an ARM. Compare the initial monthly payments for these loans.

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4.2 Borrowing: How much car can you afford?

▶ **Solution:** principal = \$200,000 and $t = 360$ months.

□ **Fixed-rate:** $r = \frac{0.066}{12} = 0.0055$. The monthly payment formula gives:

$$\begin{aligned}\text{Monthly payment} &= \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)} \\ &= \frac{\$200,000 \times 0.0055 \times 1.0075^{360}}{(1.0055^{360} - 1)} = \$1277.32\end{aligned}$$

□ **ARM:** $r = \frac{0.06}{12} = 0.005$. The monthly payment formula gives:

$$\begin{aligned}\text{Monthly payment} &= \frac{\text{Amount borrowed} \times r(1+r)^t}{((1+r)^t - 1)} \\ &= \frac{\$300,000 \times 0.005 \times 1.005^{360}}{(1.005^{360} - 1)} = \$1199.10\end{aligned}$$

Chapter 4 Personal Finance: **Chapter Summary**

- ▶ **Savings:** simple interest or compound interest

- ▶ Formulas: simple interest earned

- period interest rate

- balance after t periods

- APY

- Present value or Future value

- Number of periods to double

- ▶ **Borrowing:** an installment loan

- ▶ Formulas: Monthly payment

- Amount borrowed

- ▶ Fixed-rate mortgage vs. ARM



Chapter 4 Personal Finance: **Chapter Summary**

- ▶ **Saving for the long term: Build the nest egg (Annuity)**
 - ▶ Formulas: Balance after t deposits
 - Needed deposit
 - Monthly annuity yield
 - Nest egg needed
- ▶ **Credit cards**
 - ▶ Formulas: Amount subject to finance charges
 - Balance after t minimum payments
- ▶ **Inflation, taxes, and stocks**
 - ▶ Understand CPI, taxes, DJIA

